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Dynamic compression of elastic and plastic cellular solids

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We report the results of an experimental investigation into buckling in elastic and plastic cellular materials under dynamic compression. The buckling instabilities are in the form of a global pattern switch where the square array of circular holes is transformed into a set of orthogonal ellipses. Properties of the instabilities in the elastic and plastic cellular materials are compared and contrasted. The case of the elastic structure is considered as a delayed pitchfork bifurcation. On the other hand, the response of the plastic lattice is complex, and an irreversible global instability is only found above a critical compression rate. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4824845]

Low density cellular solids are common in nature with examples at the macroscale of cancellous bone[1] with high strength-to-weight ratio to the intricate micro-structures on the wings of butterflies which give them their iridescent colours.[2] The strength-to-weight ratio and optical properties of such materials have been exploited in the design of complex technologically important structures ranging in scale from spacecraft to photonic crystals. When these materials are compressed they may undergo elastic instabilities where the cell walls of the matrix bend or buckle. The influence of this on the global properties of the material is a current area of research. Applications include energy absorption by the crushing of layers[3] to the control of bandgaps of photonic crystals[4] and the wetting of surfaces.[5]

An interesting example of an elastic instability in a cellular material is provided by the compression of a two-dimensional elastomer with a square lattice of holes. The phenomenon is the main focus of this paper and is illustrated in Fig. 1 where it can be seen that the initially square array of circular holes is transformed into a set of orthogonal ellipses under 8% vertical compression. Note that the sides of the sample shown in Fig. 1 move inwards under compression which is a feature of the negative Poisson ratio of this cellular material.[6,7] This instability was predicted theoretically[3] and realised experimentally[8] and has been shown to exist in a variety of soft materials under compression.[9] The aim of this study was to investigate the effects of dynamic compression on the buckling instabilities present in elastic and plastic cellular lattices. In elastic lattices, the switch in states is reversible, repeatable, and global and has now been found at the nanoscale.[4,10] Euler buckling is central to the instability and is of current interest in other aspects of nanomaterials.[11] Since the pattern switch is reversible in the elastic lattice, one sample was used for all compressive testing conducted throughout this investigation. The global pattern switch also occurred in plastic samples; however, different samples had to be used for each experiment as the plastic deformation process was irreversible. The occurrence of the pattern switch in plastic materials suggests that similar buckling instabilities may result from the dynamic compression of other hard materials.

The buckling of an elastic cellular material with a square array of circular holes can be considered as an example of a pitchfork bifurcation[9] where in the case of the pattern switch the two branches of the pitchfork are buckled states which are shifted in phase by half a wavelength. When a parameter is swept through such a bifurcation there is a delay in the onset of the instability[12] and the amount of the delay will scale as the square root of the sweep speed[13] over a small range of the parameters.

The elastomer sample was made from the addition curing silicone rubber Sil AD Spezial (SADS), supplied by Feguramed GmbH. The cured material has the manufacturer's quoted value of the Young's modulus of $E_s \simeq 400$ kPa which is in accord with measured values.[14] The manufacture of the sample involved mixing equal measures of two fluids, placing the individual component fluids under vacuum to remove dissolved gases and allowing the mixture to set for an hour to ensure full curing. The mixture was poured into a purpose-built mould of machined cylindrical pillars arranged on a square lattice. The cured sample was removed from the mould, and the two side walls were cut from the sample, leaving seven columns of eight holes, flanked by a column of eight semi-circles on either side. The elastomer sample comprised a lattice of circular holes of diameter 8.79 ± 0.09 mm arranged on a square. The void fraction was 0.65. The height of the sample was 77 ± 0.1 mm, width 77 ± 0.1 mm, and thickness 7 ± 0.1 mm.

The plastic samples were fabricated using a commercial 3D printer (3D Touch triple head, Bits From Bytes) using acrylonitrile butadiene styrene (ABS) plastic. This is a lightweight rigid polymeric material with a Young's modulus of 2.3 GPa which is a factor of ∼5750 stiffer than the silicone rubber. The printer forms plastic objects by a process known as fused filament fabrication; in essence, a bead of molten thermoplastic is extruded from a hot nozzle which is moved under computer control in the horizontal direction with defined steps in the vertical to build up the object, layer by layer. It is based on the open-source RepRap project.[15] A base layer of polylactic acid was used to help prevent the printed object sticking to the print bed. Since the sample is

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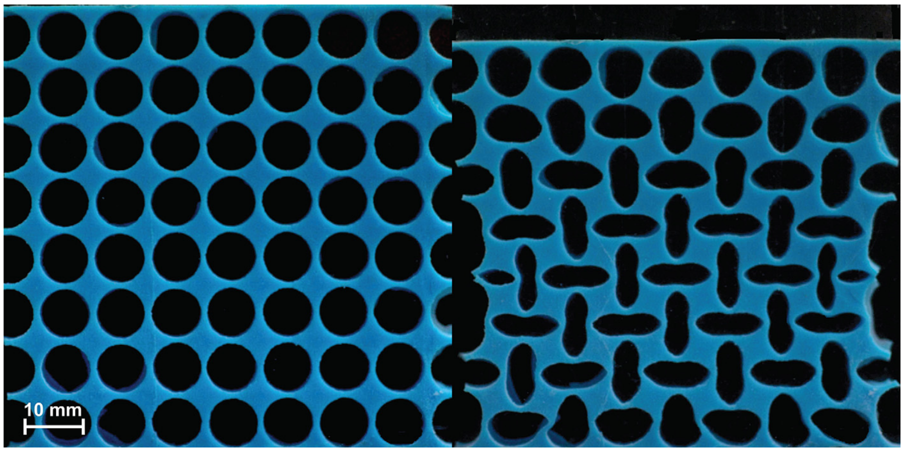

FIG. 1. The pattern switch in a two-dimensional elastomer. (a) Initial state (b) at 8% compression. The global nature of the transformation is evident apart from edge effects at the top and bottom surfaces.

built up in horizontal layers, the material properties are anisotropic; specifically, the joins between layers are weaker than the layers themselves. However, as we are primarily interested in 2D distortions of the structures, printing the samples with the plane of interest horizontal ensured that the material properties were homogeneous in the plane of interest for our experiment.

The plastic samples comprised 5×5 lattices of 9 ± 0.02 mm diameter holes arranged on a square array. The samples had a void fraction of 0.64. They were 60 ± 0.02 mm high, 60 ± 0.02 mm wide, and 10 ± 0.02 mm thick. The side, top, and bottom boundaries were all flanked by semi-circles, i.e., the samples consisted of 5×5 lattices bounded by semi-circles as shown in Fig. 4. The testing of the plastic samples was irreversible, and a new sample was used in each experiment.

The samples were tested on two Instron machines, a twin arm model 5569 for the plastic and single arm model 3345 for the rubber. Each of the samples was placed in a housing and compressed with a Perspex loader of width 100.10 ± 0.05 mm, thickness 9.68 ± 0.09 mm attached to an aluminium clamp. The housing ensured the samples remained upright and consisted of an aluminium U-shaped frame and base, which was attached to each Instron, and a front and back plate both made from Perspex with a spacing of 10.1 ± 0.1 mm. The front and back plates enabled visualization, prevented out-of-plane buckling and were removable to allow access to the experimental sample. There was a clearance of 0.7 ± 0.1 mm between the loader and the housing when the setup was assembled.

Compression tests were performed on both machines using 1 kN load cells. The sample faces were dusted with flour for the elastomeric structure and coated with vaseline for the plastic lattices to reduce frictional effects. All surfaces were made parallel to ensure even loading of the sample and prevent the loader touching the outer plates of the housing. The load associated with the displacement was recorded once every 100 ms and used to produce a stress-strain curve for the compression process.

The stress-strain data for the elastic lattice shown in Fig. 2(a) has the same characteristic form for all strain rates. The elastic regime where stress increases in proportion to applied strain is followed by a stress plateau where stress is independent of strain. Buckling of the ligaments of the cellular structure is initiated at the turnover point and grows as the square root of the strain in accord with the generic behaviour of pitchfork bifurcations.[9] These curves contain the generic features found in cellular materials under compression of a

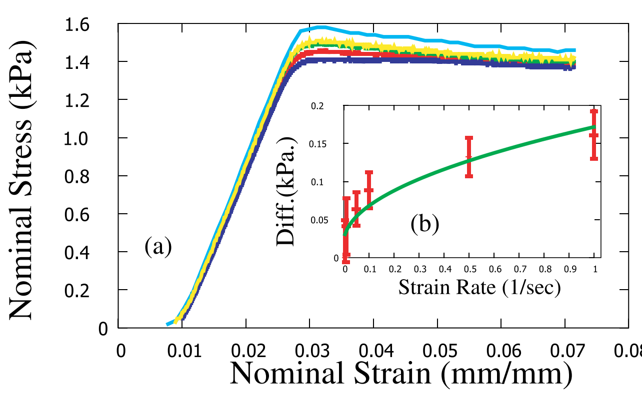

FIG. 2. (a) Plot of the stress-strain diagram for an 8×7 elastic lattice for linear compression rates at 0.005, 0.001, 0.05, 0.1, 0.5, and 1 mm s^{-1}. The stress plateau increases monotonically with strain rate. (b) Differences between the quasistatic critical point and the stresses at onset of the instability at the strain-rates used in (a). The solid line is the least squares fit of a square root function to the data. The quasistatic value was measured at a strain rate of 0.0001 mm s^{-1} which has been shown previously[8] to give a good approximation to the quasistatic limit.

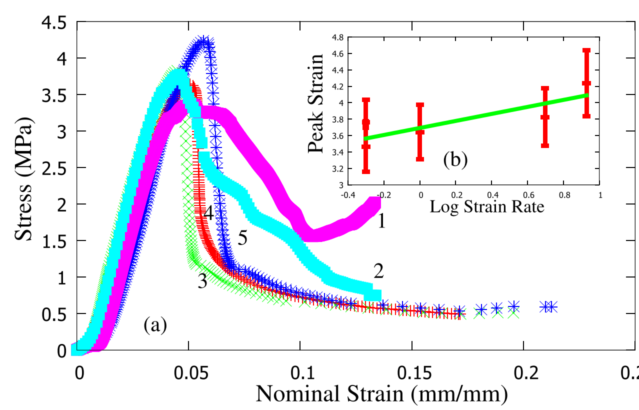

FIG. 3. (a) Stress strain datasets for the ABS plastic material which were taken at 1: 0.016 mm s^{-1}, 2: 0.05 mm s^{-1} (no stress plateau), 3: 1.0 mm s^{-1}, 4: 5.0 mm s^{-1}, and 5: 8.5 mm s^{-1} (single stress plateau). (b) Inset of peak stress for cases where the pattern transformation occurred plotted as a function of the log of the strain rate. The fitted line in (b) has a slope of 0.427. N.B. The stress strain datasets taken at 0.5 mm s^{-1} have been omitted from (a) to clarify the diagram.

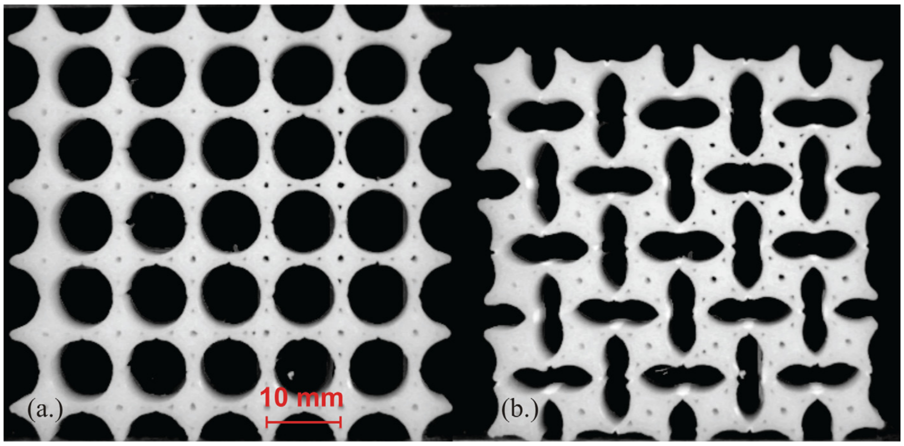

FIG. 4. Images of the compression of a plastic sample. (a) Initial state (b) after compression by 10% at $1.0\,\mathrm{mm\,s^{-1}}$. In this case the Poisson's ratio was found to be $\simeq 0$. Small ($\sim$0.8 mm diam.) holes can be seen at the center of each interstitial. These are features of the prototyping process, and their location was set to the centers of the interstitials to minimize their effect on the strength of the material.

linearly elastic region followed by a stress plateau above a critical strain.[1] In the case of a square lattice of holes, a pattern switch from an array of circular holes to one of orthogonal ellipses as shown in Fig. 1 occurs at the onset of the stress plateau.

The linear relationship between macroscopic stress and strain results from the collective microscopic elastic bending of individual ligaments. As can be seen in Fig. 2(a) there is very weak dependence of the slope on strain rate over the range investigated. This result is in accord with those reported in Ref. 16 where independence of this slope was found over a much wider range of strain rates than investigated here. The elastic lattice has the characteristics of a type I material[17] such that there is a gradual onset of the instability with increasing load which is typical of columnar buckling. Hence the plateau corresponds to buckling of the cell walls which occurs above a critical strain.

We show in the inset to Fig. 2(b) a plot of the difference between the values of the critical stress at the onset of the pattern switch, i.e., the values of the stress at the turning points in Fig. 2(a), and the quasi-static case for the strain rate range $0.005\,\mathrm{mm\,s^{-1}}$ to $1\,\mathrm{mm\,s^{-1}}$. The solid line is the least squares fit of a square root function to the data. This result is consistent with the generic delay in pitchfork bifurcations found when the control parameter is swept through the bifurcation point at increasing speeds.[13]

The stress-strain results shown in Fig. 3(a) were obtained using the plastic material. Dynamic compression above a strain rate of $0.5\,\mathrm{mm\,s^{-1}}$ results in a pattern switch comparable to that of the elastomeric structure. An example of a plastic lattice before and after compression at a strain rate of $1\,\mathrm{mm\,s^{-1}}$ is shown in Fig. 4 and corresponds to stress-strain dataset labelled 3 in Fig. 3. Compression at a strain rate below $0.5\,\mathrm{mm\,s^{-1}}$ results in localized collapse of the plastic structure and the formation of a shear band (see Fig. 5). Although distinct dynamic regimes exist above and below a strain rate of $0.5\,\mathrm{mm\,s^{-1}}$, the threshold exhibits sensitivity to imperfections; one sample compressed at $0.5\,\mathrm{mm\,s^{-1}}$ pattern switched whereas a second sample compressed at the same rate displayed localized buckling. The sensitivity to imperfections means that it is difficult to give a sharp estimate of the critical strain rate. Intriguingly, previous work on the compression of elastic lattices showed that the effect of imperfections on the quasistatic onset of the pattern switch to be surprisingly small.[14]

Above the critical strain rate, the global pattern switch is a robust phenomenon which results from a highly nonlinear event. The pattern switch is caused by plastic collapse and is hence irreversible. The stress-strain data obtained from samples compressed with a strain rate above $0.5\,\mathrm{mm\,s^{-1}}$ (see lines labelled 3–5 in Fig. 3) have the characteristics of a type II material with a linear elastic region and a pronounced peak at a catastrophic buckling transition,[17] followed by a subsequent stress plateau. The peak stress increases with strain rate as shown in Fig. 3(b) where the straight line fit to the logarithmic data indicates that the scaling is $\approx$0.427, i.e., significantly faster than the values of 0.25 found for the elastomer, i.e., the plastic switch is not an example of a simple pitchfork bifurcation. The stress plateau appears to be independent of strain rate, as the level attained by different samples compressed at different rates is the same to within experimental error, which suggests that the pattern switching process is a robust feature of the dynamic compression of plastic lattices and that the energy absorbed by the material is independent of strain rate.[1]

The location of the shear band which forms for compressive strain rates below $0.5\,\mathrm{mm\,s^{-1}}$ is dependent on local imperfections in the sample. The stress-strain results obtained

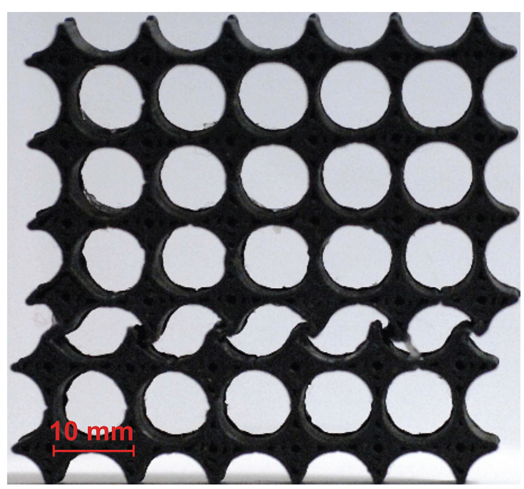

FIG. 5. An image of a localized shearband formed in an ABS sample at a compression rate of $0.0033\,\mathrm{mm\,s^{-1}}$. The location of the band is dependant on local imperfections in the sample.

for strain rates less than $0.5\,\mathrm{mm\,s^{-1}}$ have complicated forms and lack the stress plateau indicative of a pattern switch; two datasets, obtained at strain rates of $0.016\,\mathrm{mm\,s^{-1}}$ and $0.05\,\mathrm{mm\,s^{-1}}$, are included in Fig. 3.

In conclusion, a global pattern switch occurs for all strain rates in an elastic cellular solid which contains a square array of circular holes. The linear section of the stress-strain relationship does not depend on strain rate, but the amplitude of the stress plateau formed when the pattern switch that occurs increases in proportion to the square root of the strain rate. This finding is in accord with generic features of dynamic pitchfork bifurcations.[12] Geometrically equivalent cellular structures made from ABS plastic have a global pattern switch above a critical strain rate. An interesting feature which emerges is that the plateau stress is independent of the strain rate indicating that the energy absorbed by the material is also independent of strain rate. This result suggests that other rigid materials may undergo a pattern switch under dynamic loading. Several attempts were made with a copper cellular material and partial success was achieved. However, we were unable to reach large dynamic strains with the available testing machines and encourage others to test this possibility with more appropriate apparatus.

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[1] L. J. Gibson and M. F. Ashby, *Cellular Solids, Structure and Properties* (Cambridge University Press, Cambridge, 1997).
[2] P. Vukusic and J. R. Sambles, Nature **424**, 852 (2003).
[3] N. Triantafyllidis, M. D. Nestorovic, and M. W. Schraad, J. App. Mechs. **73**, 505 (2006).
[4] Y. Zhang, E. Matsumoto, A. Peter, P. Lin, R. Kamien, and S. Yang, Nano Lett. **8**, 1192 (2008).
[5] S. Singamaneni and V. V. Tsukruk, Soft Matter **6**, 5681 (2010).
[6] R. Lakes, Science **235**, 1038 (1987).
[7] K. Bertoldi, P. M. Reis, S. Willshaw, and T. Mullin, Adv. Mater. **22**, 361 (2010).
[8] T. Mullin, S. Deschanel, K. Bertoldi, and M. C. Boyce, Phys. Rev. Lett. **99**, 084301 (2007).
[9] T. Mullin, S. Willshaw, and F. Box, Soft Matter **9**, 4951 (2013).
[10] J. Li, J. Shim, A. Deng, A. Overvelde, X. Zhu, K. Bertoldi, and S. Yang, Soft Matter **8**, 10322 (2012).
[11] S. M. Carr and M. N. Wybourne, Appl. Phys. Lett. **82**, 709 (2003).
[12] P. Mandel and T. Erneux, J. Stat. Phys. **48**, 1059 (1987).
[13] R. Kapral and P. Mandel, Phys. Rev. A **32**, 1076 (1985).
[14] S. Willshaw, "On pattern-switching phenomena in complex elastic structures," Ph.D. thesis (University of Manchester, 2012).
[15] R. Jones, P. Haufe, E. Sells, P. Iravani, V. Olliver, C. Palmer, and A. Bowyer, Robotica **29**, 177–191 (2011).
[16] B. Song, W. Chen, D. Song, N. Winfree, and J. Kang, Int. J. Impact Eng. **31**, 509–521 (2005).
[17] C. R. Calladine and R. W. English, Int. J. Mech. Sci. **26**, 689 (1984).