MSc Projects (MATH61000)

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Course homepage (including these slides):
...teaching/MScProjects/index.php

MSc Pure Mathematics and Mathematical Logic

The University of Manchester, Department of Mathematics

1. Content of this module - overview

1. This module is running throughout the taught component, i.e., from the end of September to the end of May.
2. It is compulsory and worth 30 credits. (All other taught modules in the MSc are worth 15 credits, the dissertation is worth 60 credits.)
3. The unit "credit" indicates the total number of hours students are expected to work, including lectures and revision:

$$1 \text{ credit} = 10 \text{ hours.}$$

4. All material that I provide for this module (including these slides) are posted on the module's homepage. There is nothing on "Blackboard"/My Manchester.

1. Content of this module - brief outline

Part A: Mathematical writing, presentation and mini-project

This part is worth 10 credits and runs until Friday of week 8. You are expected to work 12 hours per week in Part A. Contents of this part:

- Six lectures on research skills (writing and presenting mathematics, LaTeX, resources) in weeks 0, 1 and 2.
- Students write a mini-project over a period of 6 weeks. Submission deadline: 3pm on Friday of week 7.
- Presentation of the mini-project by the students in week 8.

A detailed timetable may be found at the module's homepage.

Part B: The main project

This part is worth 20 credits and runs from Monday of week 9 to the first day of the exam period in May. The main project is supervised by some member of staff.

- You need to find a project before the end of week 6. This is done as follows:

  Please choose at least two supervisors with topics from ProjectsMATH61000.pdf and send your choices to the lecturer via email. Please indicate your preference (or say that you have no preference); we try to accommodate everybody's choice. You may contact prospective supervisors to get further information about the advertised project.
- The mini-project and the main project do not need to be connected, you can use the mini-project also to close some gaps that you see from your UG studies. However, if you want to get some inspiration for the mini-project from the available main projects, have a look at ProjectsMATH61000.pdf. An example of a main project may be found in MainProjectExample.pdf.

2. Homework to be done September 23 – 25

1. Install LaTeX (more on this below) and typeset an example. For example, write one page where you state and prove that $\sqrt{2}$ is not a rational number, or take a mathematical text book and try to reproduce one page from that book.
2. Choose a mini-project. More on this below.
3. You will have an individual meeting (30 minutes) in week 1, where we discuss the choice for the mini-project. To do so visit our poll (available after the first lecture) and select your slot as soon as possible.
4. **Important:** Please suggest at least one topic title to me via email, before the individual meeting.

2. Homework to be done September 23 – 25

4. **Important:** Please suggest at least one topic title to me via email, before the individual meeting.
   Your topic selection should be made on the basis of the following information.
   - List of possible topics, or alternatively suggest your own topic.
   - An example of how the mini-project could look like: MiniProjectExample.pdf. (Feel free to use the LaTeX-file MiniProjectExample.tex used to produce this example as well as the file MiniProjectExample.bib holding the bibliographic resources.)
   - Read the section on how to get resources of the document at hand.
   - Have a first glimpse at the available main projects

3. Resources for mathematics

Trusted resources

1. The University of Manchester Library: https://www.library.manchester.ac.uk/search-resources/? (Needs your University credentials.) I also recommend going there and browsing through text books relevant to your topic.
2. MathScinet: https://mathscinet-ams-org.manchester.idm.oclc.org/mathscinet/search.html (Needs your University credentials.)
   This is a world wide leading web-service about mathematical texts (books and research articles) that have been peer-reviewed. It is a principal and reliable resource:
   - It has reviews of the items which is sometimes telling in an understandable way what is in a particular text.
   - You can search the full text of these reviews (and title / authors).
   - The items are linked, i.e. you can search who has quoted a particular book or article, which can be very helpful to find the correct resource.

3. Resources for mathematics - trusted sources

3. Zentralblatt: https://zbmath.org/ This is a competitor of MathScinet, not as complete, but has some jewels and sometimes other or better reviews, which could be informative.
4. Ask supervisor: Via email, or by showing up during their office hours (check the supervisor's homepage or ask the reception in ATB), or by making an appointment (via email).
5. Read references of nearby topics. For example if you choose to write about a specific topic in Analysis, grab a big standard work on Analysis and look through the references. Or find a research article about the specific topic and check their references for general books about your topic.
6. Find a good book that uses your topic, but is not mainly about your topic. For example: There are plenty of books about category theory, but these are for most purposes too massive to get a quick introduction to the fundamental part of the theory. Now, the book "Algebra" by Serge Lang uses category theory to some extent and has a short and concise appendix on category theory focussing on the essentials.

3. Resources for mathematics - Preprints and unpublished notes

- ArXiv: https://arxiv.org/ (preprint server covering most sciences)
- HAL: https://hal.archives-ouvertes.fr/ (another preprint server).
- Websites of mathematicians having unpublished notes, or Lecture notes of undergraduate courses.

These sources are in principle trustworthy, but have not been refereed or reviewed. So care has to be taken. For notes of lectures, make sure the source is genuine. For example, suppose you find this source. It is not available on any website, but still the bots find it. Would you trust it?

3. Resources for mathematics - Unverified sources

The following can be used, but mainly as an entrance point, for example in order to find references to trusted sources or keywords and phrases that might help you to find such a source. You should avoid making statements in written mathematics that are only backed by unverified sources.

- Wikipedia has a large range of mathematical topics, definitions and explanations. You can start reading about some topic there if you cannot find a book right away. However, care has to be taken, frequently the material on Wikipedia is outright wrong. Still, you might get an impression on what a certain notion is about or to which part of mathematics it belongs to. Further, even if the text is wrong, the page might give you references for further search, or keywords that you can use for further search.
- MathOverflow and Stackexchange are forums where people are asking and answering mathematical questions (there are many others of course but these two are to some extent moderated, i.e. they do not contain arbitrary nonsense). Users frequently post links to trusted references, so this is of use.

3. Resources for mathematics - Unverified sources

- The internet. If you have a question and no clue how to start searching for an answer, you could write your question in some search engine and see what you get. The goal then is to finally find some trusted source.

There are two reason for finding resources:

- Find statements, proofs and explanations that you will reproduce and elaborate on.
- Give references, in order to acknowledge the original author of the result, or to quote a result that you need in your project but for which you do not give a proof. In the latter case the resource must be in the trusted category above.

  Important: References need to be available to the reader and stable (contents of websites are normally unstable, here is an exception).

4. Topics for the mini-project I

Below is a list of suggestions for mini-project topics. You can work on exactly the topic as named, or you can use the information under the Wikipedia link as an entrance point to find a related topic.

If you want to work on a topic that is not listed, you are welcome to do so. In any case we will discuss your choice in the individual meeting in week 1. The purpose of this meeting is to make sure your topic is doable in 6 weeks.

4. Topics for the mini-project II

The topics are of varying difficulty and volume with respect to prerequisites. Note that the focus in the mini-project is more on the presentation side rather than on the mathematical side.

Be careful: A seemingly easier topic might be harder to write on, because you need to write about a broader context and make a wise selection of material. A harder topic is mathematically more challenging but easier to organise.

For example, consider the two topics "Introduction to Modules" and "Modules over Principal Ideal Domains".

Obviously the second one will need knowledge of the first one. Now in the second topic one can treat the introductory material very briefly in a preliminary section or even just treat it with references. Then the project would essentially elaborate on the main theorem about modules over principal ideal domains. On the other hand "Introduction to Modules" requires a clever choice of selection (a full introduction would contain hundreds of pages) and organisation of material.

4. Topics for the mini-project III

Topics

- A. Algebra
  1. Noetherian rings and Hilbert's basis theorem. Wikipedia
  2. Cayley-Hamilton for rings and applications (e.g. Nakayama's lemma). Wikipedia,
  3. Algebraic and integral elements. Wikipedia
  4. Introduction to modules. Wikipedia
  5. Modules over principal ideal domains. Wikipedia
  6. The p-adic numbers. Wikipedia
  7. Absolute values on $\mathbb{Q}$. Wikipedia
  8. The algebraic closure of a field. Wikipedia
  9. Introduction to categories and functors. Wikipedia
- B. Metric spaces and topology
  1. Introduction to topology. Wikipedia
  2. Alexander's subbase lemma.Wikipedia
  3. Tychonoff's theorem. Wikipedia
  4. The contraction mapping principle and applications. Wikipedia
  5. Banach Fixed Point Theorem, the general statement. Wikipedia
  6. Urysohn's lemma. Wikipedia

4. Topics for the mini-project IV

7. Partition of unity. Wikipedia

C. Analysis
   1. Constructions of the field of real numbers. Wikipedia
   2. The implicit function theorem. Wikipedia
   3. Power series, formal and convergent. Wikipedia
   4. The Stone-Weierstraß Theorem. Wikipedia
   5. The fundamental theorem of ordinary differential equations: Picard-Lindelöf. Wikipedia
   6. Introduction to Banach spaces. Wikipedia
   7. Transcendence of the Euler number. Wikipedia
   8. Introduction to measures. Wikipedia

D. Geometry
   1. Helly's theorem on convex sets with applications. Wikipedia
   2. Carathéodory's theorem on the computation of the convex hull and its generalizations. Wikipedia
   3. Introduction to manifolds. Wikipedia

E. Combinatorics
   1. Ramsey's theorem and its applications to combinatorics and number theory. Wikipedia

4. Topics for the mini-project V

2. The Schröder-Bernstein Theorem and cardinal numbers. Wikipedia

F. Mathematical Logic
   1. Soundness and completeness theorem of first-order predicate logic. Wikipedia
   2. Ultraproducts and Łoś's theorem. Wikipedia
   3. The Lemma of Zorn and its applications. Wikipedia

5. Assessment

1. The marking rubric for the MSc dissertation may be found here. There is no oral presentation attached to the dissertation.
2. In the main project, 90% of the marks will be allocated using essentially the same marking rubric as in item 1 above, and 10% of the marks will be allocated to an oral presentation about the main project at the end of semester 2.
3. In the mini-project, 80% of the marks will be allocated using essentially the same marking rubric as in item 1 above, and 20% of the marks will be allocated to an oral presentation about the mini-project in week 8 of semester 1.

   However, there is one major difference in the marking of the mini-project compared to the main project: The level of the content is essentially ignored in the mini-project. Hence a harder topic will not get more marks because of its level of difficulty.

6. Where to start writing LaTeX?

I recommend working with the following components.

1. The **LaTeX program** itself.
   I recommend MikTeX. This works equally well under Windows, Linux/Ubuntu and Mac. Installation is very easy, instructions can be found on the program's website.
2. An **editor** to typeset your project.
   I recommend TeXStudio (for all platforms). Installation is very easy, instructions can be found on the program's website.

There are many good LaTeX-editors (and one major other LaTeX distribution, called TeXLive, or MacTeX for the Mac) available. The choice above seems to me the simplest one to get started. (Note that not every good editor has explicit support for LaTeX.) Let me know if you cannot get these to work.

6. Where to start writing LaTeX?

There are many resources on the web to learn LaTeX, here are a few:

- LaTeX homepage,
- LaTeX Wiki book
- Paul Johnson's LaTeX site
- TeX FAQ
- CTAN: Contains (almost) all available packages.
- Find (almost) any symbol in the Comprehensive LaTeX Symbol List
- A rich source of examples of diagrams and pictures, including code, may be found at http://www.texample.net/tikz/examples/

Most LaTeX editors will also assist you finding LaTeX-documentation and will suggest various standard templates for writing the most common pieces of mathematics (like lists, theorem and proof environments, displayed formulas, matrices, diagrams).

7. How to find a suitable topic?

Using the material mentioned in the homework section you should first narrow down your selection to a number of topics that are appealing to you, say you have narrowed it down to 5 possible topics. Find trusted sources (download those that are accessible and then go to the library). Once you have the resources in place you should take into account the following:

- The mini-project is supposed to be between 10 and 13 pages (including cover sheet, references and index). Hence the mathematical text (including the introduction) should be between 7 and 10 pages. Can you write about your selected topic in 7-10 pages?

7. How to find a suitable topic?

- How much of the topic is new to you? From the 60 hours allocated for the mini-project, you shouldn't spend too much time in learning new material. If you have never written a project and you have no experience with writing LaTeX, I recommend choosing a topic that you know more or less already, or you can fill the missing details in 10 hours. If you have already written some mathematical text using LaTeX you might want to allocate at least 20 hours for the actual writing, a safer option would be 30 hours.
- You will have to present the material in week 8 to the lecturer. Can you see yourself talking about the topic? (In week 2 there will be a lecture on presenting mathematics.)
- Can you find a good variation of trusted resources from which you can choose? It is not a good idea to just have a single source as you need to produce your own account of the topic.

7. How to find a suitable topic?

- You will need to say *something* in the mini-project. Which idea will you communicate?
- Are there interesting applications of the main topic that you can elaborate on?
- You need to include examples. Do the sources have helpful examples?
- Can you say something about the wider context of your topic?
- How systematic do you want to be? Do you want to develop some theory in the first place, or explain a specific theorem? Both tasks have their own challenges.
- Whom will you be writing for? Is the text to be understandable for a year 1 UG mathematics student? A year 3 UG mathematics student? You will need to write about what a reader is supposed to know to read your project.

7. How to find a suitable topic?

Every project should have an introduction and a list of references. It could also contain other sections. For example (we will talk about these next week in more detail):

1. **Introduction**
2. Preliminaries and/or background
3. **The body of the text**
4. Applications
5. Further Reading
6. Conclusion
7. **References**
8. Index

There are various books about how to write mathematics. There is no one way, but there are some guiding principles. More on this next week. For now, maybe read the preface of [Viv14].

7. How to find a suitable topic?

A summary of how to write mathematics that I very much agree with is given by Paul Halmos in [Hal70, p. 124, last paragraph of section 1]:

> *"The basic problem in writing mathematics is the same as in writing biology, writing a novel, or writing directions for assembling a harpsichord: the problem is to communicate an idea. To do so, and to do it clearly, you must have something to say, and you must have someone to say it to, you must organize what you want to say, and you must arrange it in the order you want it said in, you must write it, rewrite it, and re-rewrite it several times, and you must be willing to think hard about and work hard on mechanical details such as diction, notation, and punctuation. That's all there is to do."*

8. Global strategies for writing

1. Communicate an idea, hence you must have something to say. Ideally it should be one main idea, not a conglomeration of loosely connected facts. If you are not writing with a single main statement in mind, identify a coherent theme.
2. You must have somebody to say it to. Who is your audience? This is very important because it will give you a separation of the material that you will elaborate on and the material that will be in the prerequisite section (or will just be quoted). It will also help you make a decision on what to justify and what to deem as "known".
3. When the first two points have an (approximate) answer: make a rough plan how to organise the material. Do you want to write about exciting applications of your main theorem or theme? How much additional theory, formalism or explanations will these applications need to address the target audience?

8. Global strategies for writing

4. In order to actually start writing: write something!
   It does not need to be the first section and it should not be the introduction (the introduction is written at the very end). For example you could try to write up one of the main theorems that you want to communicate, including a (sketch of a) proof. While doing so, you will see material that has to be included, objects that have to be defined and other explanations that need to be present. This gives some information about the organisation of material.
5. Write sections in spirals: 1,2,1,2,3,1,2,3,4,... and re-organise if necessary. Look at earlier sections in the light of new material that has been added. Ask yourself if the original organisation needs amendment. (Normally the answer is: yes).
6. A common problem in writing mathematics is that at the beginning you do not foresee all the technical material that you have to include, which forces a rearrangement of the original plan.

8. Global strategies for writing

Recall the following possible structure of a project, which might help in making a plan: The project should have an introduction and a list of references. It could also contain other sections. For example:

- **1. Introduction**
- 2. Preliminaries and/or background
- **The body of the text**, having sections. In this example, say it has sections 3, 4, 5 and 6.
- $\vdots$
- 7. Applications
- 8. Further Reading
- 9. Conclusion
- **References**
- Index

There are various books about how to write mathematics. There is no one way, but there are plenty of guidelines available, for example see [Gil87; Hal70; Knu89; Kra97; Ros06; Viv14].

9. Writing principles

1. Everything you say needs to be justified, by either a proof, a reference, or some form of indication why a statement is true. For example, words like "clear", "obvious", etc. contain your judgement that the statement is left to the reader and you assume that they can fill the gap without a problem.

   There are certainly limits to this. For example in calculations you do not need to justify standard algebraic operations (binomial formula), assuming your audience has basic mathematical education.
2. Think of correct notations early on and try to follow standard conventions. Consult multiple sources if necessary.
3. Write a summary at the beginning of each section.
4. Create subsections if this helps to organize the material.

9. Writing principles - numbering and notions

5. Numerate statements for easy referencing and number them sensibly. For example, if you have an item numbered by "3.2", there should not be another item with the same number: Hence, if you have no subsections in section 3 say, all facts in section 3 should have two digits. However, if you have subsections in section 3, all facts in section 3 and its subsections should have three digits (see the mini-project example for implementation using the command \numberwithin).
6. When you introduce a new notion, make sure the reader can find this easily later on. Traditionally you will create a "Definition", but sometimes new notions are introduced within facts or in a free floating text. When you use the notion later on, help the reader finding the definition easily. The easiest way is to **boldface new notions**, wherever they are defined. An index is obviously also very helpful and very easy to do, see the mini-project example for implementation.
7. Many mistakes in projects come from ill-defined objects. Make sure that all objects that you are introducing are well-defined.

9. Writing principles

8. Repeat notions and facts that have not been used for a while by linking to earlier places.
9. Avoid using unspecified references, like "As we have seen *above*". If this on page 10, where does "above" point to? Instead write a precise reference or describe the place that you are referring to. For example: "By the remark after lemma XXX", or "at the beginning of the proof of Proposition XXX".
10. Use of general conventions. If you are working under some specific assumption, e.g. "all rings are commutative", repeat all these conventions at the beginning of each sections. If you have plenty conventions, you can say "the conventions of the previous section remain in force". Conventions that are used throughout the text should be stated at the end of the introduction.
11. Think carefully about which facts you want to label as Theorem, Proposition, Lemma, Remark, Observation etc.
12. Frequently, technical material is gathered in lemmas. Say before these lemmas where they will be used.

10. Good mathematical style

Write in complete sentences, and in proper English. Each sentence should have a subject, an object and a verb.
For example,

Let C be a perfect subset of X. And nowhere dense.

is not correct. Instead you should write

Let C be a perfect subset of X. Suppose in addition that C is nowhere dense.

When you proof read, you could improve this to

Let C be a perfect and nowhere dense subset of X.

10. Good mathematical style

- Simple declarative sentences are the best for communicating definitions and facts. For example,

  Let f be a continuous function on $[0,1]$, and so it is bounded.

  is harder to read than

  Let f be a continuous function on $[0,1]$. Then f is bounded.

- Use formulations like "obvious", "clear", "can easily be seen" etc., with caution. Is the fact still clear three weeks after writing? Notice: Declaring something as obvious, or clear, contains two information: You believe the reader can fill the gap, and the fact is supposed to be justified without you giving further details.
  A minimal request when declaring something as obvious: if your supervisor asks you for details, you can fill the gap.
- Display important or complex formulas. See 2.2.6(ii) and (iii) of the mini project for examples (find the code in the LaTeX-file).

10. Good mathematical style

- Do not use verbal contractions like “isn't”, “don't”, “let's”. Instead write “is not”, “do not”, “let us”.
- You should normally write the project in the first person plural (“We show...”, “We prove...”, “We have seen that...”, etc). This is meant to include the reader in the reasoning and is the standard in mathematical writing.
- Avoid using temporal expressions (We *will* show). This normally introduces unnecessary wording.
- Use good punctuation. Sometimes a comma, or a lack thereof, can even change the truth value of a sentence. Further, displayed material, including diagrams should be terminated with a period if the displayed material ends a sentence.
- Do not start sentences with mathematical symbols and do not let symbols clash at punctation. For example

  “This shows that $x \in G$. $x \in H$, because...”

  is unreadable. Instead write

  “This shows that $x \in G$. Furthermore $x \in H$, because...”

10. Good mathematical style - resist symbols

- Do not use logical symbols as abbreviation and do not mix logical symbols and words. Do not use mathematical notation inappropriately (particularly when it is clearer to write something in plain English). For example "If $\exists x \in \mathbb{Z}$ with the property..." is wrong. Instead write "If there is an integer x with the property..." Another example: Do not write

  > *f is a continuous function on $[0,1] \Rightarrow \exists x_0 \in [0,1]$ s.t. $f(x_0) = \sup_{x \in X} f(x)$.*

  Instead write

  > *Let f be a continuous function on $[0,1]$. Then there exists a point $x_0 \in [0,1]$ such that $f(x_0) = \sup_{x \in X} f(x)$.*

- Resist symbols that are not used in statements, like "Every differentiable function f is continuous." What does f contribute here?

10. Good mathematical style

- If you quote somebody literally, display the quote or use quotation marks; furthermore give a reference to the literature and do not modify the original text in your quote. For an example see the quote of Halmos.
- Use a spell checker, but be careful: they only detect blunt mistakes. For example, no spell checker will detect the two mistakes in the sentence

  > *The equation $x^2 = a$ has a solution in $\mathbb{C}$ for very complex numbers a.*

  Also check for repeated words, like "the the" (occurs frequently as a copy and paste error); Spell checkers are implemented in most editors and can help you with the verification.

11. The role of examples

Your project should have examples and counterexamples, but you should think carefully about the purpose of your (counter-)examples:

1. An example might illustrate a definition. The purpose can be to inform the reader about major instances where this definition applies to. Or, it might connect the definition with the reader's intuition and knowledge to help them cope with the frequently abstract level of mathematical objects or concepts.
2. An example might show how a specific fact is useful in a concrete context that is known to the reader. For example, the intermediate value theorem could be followed by an example showing how the theorem establishes solvability of the equation $\cos(x) = x$ in $\mathbb{R}$.
3. A counterexample might show that perceived improvements of facts do not hold. (E.g.: After the theorem saying that differentiable functions are continuous, there could be an example showing that the derivative is not necessarily continuous).

11. The role of examples

1. A (counter-)example could explain why certain assumptions in facts are necessary. For example, the boundedness theorem (saying that every continuous function, defined on a closed and bounded interval, attains a global maximum) could be followed by two examples showing that both assumptions "closed" and "bounded" are necessary for the conclusion of the theorem. (Do not write: "The theorem fails without these assumptions", because a theorem cannot fail. Only the *conclusion* of the theorem might fail.)
2. Do not write trivial examples without a good reason. If you define what is a topology or a Boolean algebra and then your only example is "$\{\emptyset, \mathbb{R}\}$", why would you state this? One reason could be: It is the smallest topology on $\mathbb{R}$ and this might be of interest for systematic reasons. However it does not elude the definition of topology and a more serious example should be presented.

11. The role of examples

1. Avoid writing overly complicated examples or endless example calculations, unless this serves some purpose - in that case: name the purpose.
2. If appropriate you can also include a picture or a diagram. Use "TikZ" to implement this (it is a package that is already loaded in the mini-project example). See http://www.texample.net/tikz/examples/ for plenty of ready made examples including LaTeX-code, which you can copy into your LaTeX-file and then tilt according to your needs.

12. Some principles for writing proofs

1. Make clear in the proof where the assumptions are used, even if this is very obvious. This also avoids mistakes.
2. Give a structure to proofs by making claims or even subclaims, where possible. When a proof of a claim is finished, make sure the reader realizes this.
3. If you have to process various cases, use "case 1", "case 2" etc. on separate lines.
4. Longer proofs may have subclaims that are worth a lemma.
5. Very long proofs could be split up in lemmata where possible. You could also write an outline or communicate the key idea of a long proof. Another way to help the reader understand a complicated proof is to run it in a specific case, as long as the central idea is still visible in that case.
6. If you do a proof by contradiction: tell this to the reader in some form.
7. If you do a proof by induction, say what you are inducting on.

13. Referencing and citations - general rules

- Collect the bibliographic sources at the end of the project. Each item in your list of references should be cited somewhere in the main body of the text. The bibliographic information of every specific item should be complete and unambiguous.
- References in the text should be as precise as possible. If you cite a specific definition or fact, give a full reference. For example the reference [Cie97, Theorem 4.3.4, p. 53] to a book in the proof of 2.2.11 of the mini-project example is correct. Think of the reader: Will the reader *immediately* find what you are pointing to with the information in the reference, or can you be more specific?
- You can give references for several facts that you are writing about at the beginning of the corresponding section. For example by saying "The material in this section comes from Reference 1 and Reference 2".

13. Referencing and citations - implementation

- Avoid using references to websites without any other supporting evidence. If you have to quote a website, include full page title, full URL and a date when you have visited the website. For example "Prime numbers, `www.wikipedia.org`" is not appropriate. Instead write "Prime number, `www.wikipedia.org/wiki/Primes`, retrieved on 15/9/2018".

13. Referencing and citations - implementation

Use BibTeX or BibLaTeX for the reference mechanism.

- The best mechanism is to use biblatex as is done in the tex file of the mini-project example. Here is a link to the documentation
- The bibliographic information is stored in a separate “bib-file”. For the mini-project example this can be found in the bib-file for the mini-project example.
- Find bib-files on `https://mathscinet-ams-org.manchester.idm.oclc.org/mathscinet/search.html` as follows: Search for an item; if the website gives you a list of entries, open the one that you are looking for; then click on 'Select alternative format' and choose 'BibTex'. Then you can copy and paste the displayed information into your bib-file.
  There are also bib-files on arXiv if you are working with preprints.

13. Referencing and citations - implementation

- In order to quote an item that is listed in the bib-file, use the \cite - command together with a shortcut to the entry of the bib-file. For a concrete example look at the purple reference [Sto36] at the beginning of the introduction of the pdf file of the mini-project example. This is produced with the command \cite{Stone1936} in the tex-file of the mini-project example. The label "Stone1936" refers to the name of the corresponding entry in the bib-file of the mini-project example. The actual output, namely [Sto36], is created by the bibtex program.
- You can also add entries to the bib-file yourself by using the appropriate format, which may be found here.

14. The introduction

This is written at the end, when you know exactly what is in your text. In the first place, you should ask yourself the following questions: Which purpose is served by the introduction? When you read books or papers, what do **you** expect from their introduction and what do you get?
Here are some hints on what to think about when writing the introduction. Not all of the items below fit to every introduction, but they might give some ideas on how to make progress.

- Name a general context. This is often the wider subject area (or areas) to which the project belongs.
- Describe – in a non-technical way – a main idea, or theme, or result of the project. You may also use significant examples to do this.
- You could write about the motivation for the project.
- Who is addressed by the project, what is its intended audience, what is the reader assumed to know?
- At the end of the introduction, normally a brief overview of the project is given, e.g. by making a rundown section by section.

14. The introduction

- It is a good idea to include a table of contents before the introduction, to display a quick outline (which is done automatically with the command \tableofcontents just before the introduction).
- If you are describing several results, try to put weights on them, whenever possible. Say what is the main theorem and what is secondary.
- If you want to include something about the history of the subject or about mathematicians, you must include a trusted reference where this can be verified.
- If you do not have much to say about prerequisites, or applications, or further reading in the body of the text, you man say such things at the very end of the introduction.
- **Hint:** Read introductions of papers. In trusted resources you often find well written introductions. Try to extract some principles that are used in these introductions. On arXiv you might find some less favorable examples.

15. Preliminaries and/or background

If you have many prerequisites or explicit facts that you want to communicate without interrupting the main text, you could write a section on background and preliminaries. This could include:

- Notational conventions with reference to standard literature.
- What general level or basic terminology and facts are you assuming and where can the reader find these in the literature.
- Which facts are you using and where can the reader find these in the literature.

16. The Presentation

The main problem of a presenter of mathematics is what?

TIME

Here are some hints and strategies for presentations of mathematics using a board (black or white). A presentation using slides

- is harder to follow for the audience,
- is less entertaining for the audience,
- connects less with the audience than a talk on the board, where the audience sees the presenter working, and
- takes much longer to prepare.

So I will focus on presentations using a board.

16. The Presentation - Preparation

You should prepare your talk by writing notes, which you can bring to your talk. For preparation of these notes:

- Focus on the absolute essential thing you want to say about your project. If this is too complicated (e.g. because you would have to introduce too much notation), prepare a weakening of the main statement, which still has the main idea visible. One could even focus on an essential example (but this is generally hard to do, because examples many times hide the essence of an idea).
- Try to develop a central idea. Avoid putting the main idea at the very end, in fact name the concrete goal of your talk as soon as the terminology has been clarified.
  If you run out of time, the talk will either not reach the idea or you have to rush through material. It is better to plan for certain material that can be omitted. You could put weights on the material. Label those things that are absolutely necessary for the talk and those that are omittable (but still all this material should originally fit into the allocated time).

16. The Presentation - Preparation

- Define objects properly and conscientiously. Plan sufficient time for definitions and explanations thereof. If the audience has no means of understanding the central objects of your talk, you have lost them.
- Proofs: Sketch central ingredients and key ideas of proofs. If you need to do calculations: Focus on the genuine part of the calculation. Normally, you will have very little time for proofs.
- Sometimes the value of your main theorem is not clear from the statement alone. Try to convince the audience of its value or its usefulness or its beauty.

  You can use examples or talk about the insight that the main result gives. This could also be of a conceptual nature.
- Include time for interruptions. These could be: questions from the audience; you are making mistakes/typos.

$\curvearrowright$

16. The Presentation - The Talk

- Speak to the audience, not to yourself or the material that you are presenting. Look at the audience.
- Organise the board. When you need to erase the board: do you want to keep some essential information on the board?
- Use the opportunities of the board. For example you can show material in parallel (like a definition and an example thereof at the same time).
- Practice your talk with a timer. Also train the mechanics of writing with chalk or a pen and how to clean the board quickly.
- Not everything from your notes needs to be written on the board. Some people write the notes in two colors (what to write on the board, what to say only).
- Think very carefully about what to write on the board. Long sentences take time. You may use abbreviations, but make sure everybody understands these correctly.
- Look at the room a couple of days before the talk, so you know how much space you have on the board.

16. The Presentation - Using Slides

- The main problem with presentations of mathematics using slides is even more the timing. Many presentations contain way too much material that is impossible to be digestible for the audience in the allocated time.
- A second common issue with slide presentations is the dissonance of what is on the slides and what the speaker is saying. The audience tends to read what is shown to them. If the presenter is talking about something else, this is often overheard.
- A characteristically bad talk makes both these mistakes. The audience sees slides, tries to read them, but is not given enough time to do so - while the presenter is talking about something that is actually not on the slides.

16. The Presentation - Using Slides

- In order to get the amount of presented material right, prepare a standard blackboard talk. Then you roughly know how much material the audience can digest.
  After that you can include the material that you would have written on the board into the slides.
- It is hard to say how many slides can go into one 20 minutes talk because this depends how densely the slides are packed, or if you also show diagrams or photos, for example.
  A good estimate can be calculated as follows. Add up the time needed for each slide to be shown:
  1. For written text only, read out the text loud, this is the minimum time of how long the slide should be shown to the audience.
  2. For mathematical formulas: Write out the slide, this is the minimum time of how long the slide should be shown to the audience.
- If you follow this strategy, please be strict. There is obviously a trap here: One is tempted to put more on the slides to "just show the audience this or that". Then the bad part of the presentation starts.

16. The Presentation - Using Slides

Implementation using LaTeX and the `beamer` package.

Homepage of the beamer package:

https://www.ctan.org/pkg/beamer

This package should be included already in most LaTeX installations.

From the website you can also get a very detailed documentation and the code of a running example. Copy this example to an empty file, this file then should compile right away.

16. The Presentation - assessment criteria

The talk will be assessed by me using the following criteria:

- Is the mathematics correct?
- Is a context given and is a central goal communicated?
- Are all objects that are necessary for understanding the contents, properly defined/explained? This of course depends on the intended audience (so a word about that at the beginning should be said).
- Is the presentation balanced and is the material presented efficiently (inefficient/unbalanced might for example be: spending much time on something obvious and then quickly going through something difficult)
- Is the speaker talking to the audience?
- Is the handwriting legible (if you do slides: is there enough time for the audience to actually read the slides and to listen to the speaker)
- How does the speaker respond to questions?

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